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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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THE SIMPLIFYING ASSUMPTIONS, REDUCING THE STRICT APPLICATION OF CLASSICAL HYDRODYNAMICS TO PRACTICAL AERONAUTICAL COMPUTATIONS.

By Max M. Munk.

Paper read at International Congress for Applied Mechanics, Delft, Holland, April 22-28, 1924.

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THE SIMPLIFYING ASSUMPTIONS, REDUCING THE STRICT APPLICATION OF CLASSICAL HYDRODYNAMICS TO PRACTICAL AERONAUTICAL COMPUTATIONS.*

By Max M. Munk.

The application of classical hydrodynamics to the solution of aeronautical problems is based on simplifying assumptions of a fundamental nature, as the process involves setting aside the viscosity and compressibility of the air in the first place. These two properties greatly complicate any analytic treatment of aerodynamical questions, and by neglecting them it becomes possible to obtain valuable, though approximate results, which are of great practical use.

The errors introduced by neglecting viscosity and compressibility, and the corrections therefore necessary, as well as the criteria for model tests free from such errors, have often been discussed and are not the subject of this paper. But the simplifying assumptions which simply allow the application of hydrodynamics are not enough. The mathematical treatment required is still too involved and difficult for use in practice. This paper deals then with the simplifying assumptions necessary to make classical hydrodynamics adapted for practical use.

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A similar development took place in the theory of elasticity. The assumptions expressed by Hooke's Law and by others removed to a great extent the difficulties caused by the physical aspect of the problem. But even then, the mathematical treatment had to be simplified too, and it was not until the theory of infinitely elongated beams and columns had been worked out, that the theory of elasticity became a valuable tool in the hands of practical engineers.

The general method followed to simplify the numerical work in hydrodynamics consists merely in neglecting quantities of a low order of magnitude. I proceed at once to discuss how this is done in the different problems of aeronautical hydrodynamics. The solutions having found a practical application up to now are:

1. Theory of the lateral air forces on airship hulls
2. Theory of wing sections in a two-dimensional flow
3. Theory of wings with a finite span
4. Propeller theory

1. The Theory of the Lateral Air Forces on Airship Hulls.

There are earlier attempts to investigate the flow around airship hulls with circular cross-section moving parallel to their axis, the method consisting of first choosing a certain distribution of fictitious sinks and sources and then determining the shape of the hull and of the streamlines corresponding to that distribution. By substituting doublets for the simple sources or sinks, the lateral motion of airship hulls with cylindrical cross sections can be investigated in quite an analogous way. It is difficult, however, to find such a pair of distributions of sources and sinks, and of doublets which give rise to the same shape of the hull. The method is rather laborious; furthermore, it is not adapted for practical use.

Airship hulls have an elongation ratio of the length to the maximum diameter up to 10, and more, and it suggests itself to introduce the simplifying assumption of an infinite elongation ratio. This is not of so great use for the problem of longitudinal motion (parallel to the axis), since, with diminishing diameters, logarithmic terms become dominant. The hydrodynamic flow set up by the longitudinal motion is not of so great practical importance, however. It is known that the additional apparent mass of the hull in this case is small when compared with its actual mass. In many cases it can be neglected. The velocity of flow at all points is small when compared with the velocity of motion, and hence the pressure differences are small too. A blunt nose is an exception to this rule, but then, a blunt nose is in contradiction to the assumed infinite elongation, which should reduce all zones of the hull to an approximately cylindrical shape. Near the blunt nose, therefore, large air velocities and pressure differences do occur in a straight flight. Along the larger portion of the hull, however, the velocity of the air relative to the hull can be assumed to be equal to the velocity of flight.

The most important practical problem next to the drag experienced by the hull in straight flight is the computation of the lateral forces acting on the hull when flying with an inclination of the axis with respect to the direction of motion, or when flying along a curved path. The computation of these forces and of the pressure distribution giving rise to them becomes greatly simplified by the assumption that the elongation be infinite. Each zone of the hull can then be considered as cylindrical, and the component of the velocity distribution set up by the lateral component of motion can be supposed to be the two-dimensional flow around this cylinder, corresponding to the lateral velocity component. This two-dimensional flow is generally known in practical cases, the cross-section is often circular or at least approaching a circle or ellipse and the flow produced by its motion can then easily be computed. The potential of this two-dimensional flow may be denoted by Φ, and some provision may be made so as to make the potential of all cross-sections equal over all points of one surface at right angles to all streamlines. For circular cross-sections this could be the plane through the axis at right angles to the lateral component of motion. Then

$$v = \frac{\delta\Phi}{\delta x}$$

gives the longitudinal velocity near the surface set up by the lateral motion. In practice it is small when compared with the longitudinal velocity component of motion. This suggests the additional simplifying assumption that the angle α between the axis and the direction of motion be always small. Then the pressure variation, which according to Bernouilli's law contains the square of the velocity to the hull $(V + v)^2$, becomes approximately linear in v and proportional to $2\,Vv$, the term with V^2 giving rise only to a constant pressure and the term with v^2 being small of the second order of magnitude. It follows, for the main case of circular sections, where, as is known, the potential of the two-dimensional flow in question at the points of the circle is proportional to their distance from a diameter, that the pressure gradient parallel to the plane of symmetry of the flow at the points of the boundary of such a cross-section is constant.

If all cross-sections are geometrically similar, their apparent additional masses in the two-dimensional problem are proportional to their areas; with circular cylinders in particular, the apparent additional mass is equal to the mass of the displaced fluid. Hence the apparent additional mass of a very elongated hull with circular sections for lateral motion is equal to the mass of the displaced air; if the section is not circular, the apparent additional mass is k times as large, where k denotes the corresponding ratio for the section in a two-dimensional flow. It follows that the entire couple of the lateral air forces is equal to

$$V^2 \frac{\rho}{2} \sin 2\,\alpha \times \text{Volume}$$

(where ρ denotes the density of the air). (Ref. 1.)

A formula equally as simple can be found for the distribution of the lateral forces along the axis. Suppose the ship to fly straight and horizontally with the axis pitched up under an angle α with the horizontal. Consider a vertical layer of air at right angles to the plane of symmetry of the ship. When the hull passes through it, a two-dimensional flow is set up in that layer, corresponding to the lateral velocity component $V \cos \alpha$ and to the cross-section of the hull where the layer of air intersects it. The area of the cross-section, and hence the apparent additional mass of the two-dimensional flow in the layer is varying as the hull passes along with the velocity V. Hence a change of the momentum of the two-dimensional flow in the layer takes place continuously, giving rise to the reaction

$$V^2 \frac{\rho}{2} \sin (2\alpha) \; k \frac{dS}{dx}$$

where

- V denotes the velocity of flight
- α the angle of pitch
- S the area of cross-section
- k the coefficient of apparent additional mass of cross-section
- ρ the density of air
- x the coordinate along the axis of the hull.

For circular cross-sections, $k = 1$.

The same assumptions and arguments lead to useful formulas for the lateral forces on airship hulls flying in a curve. The details can be found in Ref. 2.

2. The Theory of Wing Sections in a Two-Dimensional Flow.

The theory of the wing section is in a way the two-dimensional analogy to the theory of airship hulls with circular cross-section. A large amount of literature exists about the former problem, I mention only Kutta, who originated this branch of aerodynamics, and Joukowsky, who obtained most publicity in connection with it.

The method followed by Kutta and his successors is based on the conformal transformation of the wing section boundary into a circle, a process requiring very laborious mathematical work, and which cannot be applied to most actual wing sections but must be restricted to certain simple sections distinguished by no other advantages.

In order to reduce the solution of this problem to computations to be made in the office of an airplane factory, it suggests itself to consider the wing section as infinitely elongated in analogy to the airship hull just treated. The assumptions are then that (a) the maximum thickness, and (b) the maximum camber, is small when compared with the length of the chord. These two assumptions are fairly well complied with by nearly all wing sections used in practice. In addition, it is convenient, though not absolutely necessary, to assume the angle of attack between the chord and the direction of motion to be small too. Then the velocity of the flow created by the motion of the wing is small when compared with the velocity of motion, and can be neglected when added to it. The simplification leading to a convenient development of the main formula consists now in substituting a new boundary in the problem. Instead of the boundary of the section, the chord, that is a straight line in the immediate neighborhood of all points of the section, is taken as the reference line for the conditions of flow. For the computation of the lift, for instance, the wing section can first be replaced by its middle line, having as ordinates the arithmetical mean ξ, of the upper and lower ordinates of the wing section, the chord being the axis of abscissae x. Then the velocity component of the flow at any point of the chord and normal to it is approximately $V\, d\xi/dx$ and this reduces the original problem to one the solution of which is well known. Any desired quantity referring to the flow can be expressed as a linear function of all mean ordinates of the section, either as an infinite series or as a definite integral. The latter is more convenient for practice, particularly if the chord passes through the rear edge of the section. The lift is given by the condition that the air does not flow around the rear edge; this leads to the formula

$$L = V^2 \rho \int_{-1}^{+1} \frac{\xi\, dx}{(1 - x)\sqrt{1 - x^2}} \quad \text{(length of chord = 2)}$$

The pitching moment with respect to the middle of the chord results

$$M = \int_{-1}^{+1} \frac{x\ \xi\, dx}{\sqrt{1 - x^2}} \qquad \text{(Reference \#3.)}$$

When computing the pressure distribution around the wing section, the thickness of the section can no longer be disregarded but gives rise to similar definite integrals giving terms of the same order of magnitude as do the mean ordinates. The pressure on both sides is diminished owing to the thickness and hence a section of finite thickness is supported more by suction on its upper side than by pressure on the lower.

3. Theory of Wings with Finite Span.

The practical difficulties of this problem lie in its being a three-dimensional one. As is well known, Dr. L. Prandtl attacked it with the methods existing for the investigation of three-dimensional flows, using Helmholtz vortex lines, a method which was also tried by Lanchester. In this way, Dr. Prandtl obtained valuable results, though chiefly qualitative ones. Practical computations can only be made by reducing the problem to a two-dimensional one by means of suitable assumptions. It is significant in this connection that Dr. Prandtl from the very first virtually abandoned the three-dimensional treatment by assuming the vortex lines to be parallel to the direction of flight rather than to coincide with the streamlines. The strict two-dimensional treatment of the problem requires in addition that the components of the flow set up by the wing parallel and lateral to its motion be neglected when added to the velocity of flight. Then, the use of the Helmholtz vortex lines can be avoided altogether and the usual methods for investigating two-dimensional flows can be used instead. This is a proceeding much more desirable, for the method of vortices and vortex lines seems not to appeal readily to minds not thoroughly trained mathematically, and gives rise to confusion among practical men rather than serving to enlighten them.

It should be mentioned in this connection that Dr. A. Betz investigated the air forces of a biplane cellule by combining in a particular way the wing theory and the wing section theory. Following Dr. Prandtl he assumed the actual vortex lines to be parallel; and furthermore, he replaced the wings by fictitious concentrated vortex lines, obtaining thus a continuous system of vortices. He obtained valuable qualitative results, but his method is too laborious for practice and no exact quantitative results can be expected from it. His assumptions amount to replacing the wings by cylinders of infinitely small diameter, which does not seem justified to me as the distance between the upper and lower wing of a biplane cellule is not large when compared with the wing chord. And even if it were much larger than it is, so that neglecting the chord would be permissible, it would not yet be evident that the first term, that is, the circulation term characteristic for the lift and vanishing inversely as the distance, is dominant. It seems to me that at least the second term, characteristic for the moment of the air force and vanishing inversely as the square of the distance, should be taken into account too, as it is of the same order of magnitude as the first one (References 4 and 5).

The fundamental assumption of the simplified wing theory is, accordingly, that the air contained in a plane layer at right angles to the direction of flight remains inside the same layer and moves as a two-dimensional flow. Far in front of the airplane, the layer is supposed to be at rest. While passing through it, the wings gradually built up a two-dimensional flow in it. After the wings have passed, the momentum of this flow is equal and opposite to the lift transferred from the layer to the wings. The two-dimensional flow is further determined by the condition that the impulsive pressure, necessary to create it and acting along the boundaries of the front view of the wings, is equal and opposite in direction to the distribution of the lift transferred to the wings. It can be demonstrated in particular that the two-dimensional flow has only obtained half its strength when the wings are passing the layer. This factor 1/2 finds its analogy in many other branches of theoretical mechanics.

The kinetic energy of the potential flow can be computed. The work consumed in overcoming the drag of the wings (called the induced drag) is equal to the kinetic energy transferred to the layers after the wings have passed them. The two-dimensional flow, already half created in the neighborhood of the wings gives rise also to a difference between the "effective" angle of attack (between chord and relative air flow) and the "geometric" angle of attack (between chord and direction of motion), called the "induced angle of attack."

As an additional assumption, the induced drag and induced angle of attack are generally replaced by the minimum value of these two quantities compatible with the area of the surface, the span of one wing or plan view of several wings, the magnitude of the lift, the density of the air and the velocity of flight. A further additional assumption which is often used is that the aspect ratio b^2/S, is large. There are, further, very simple rules referring to the diminution of the lift or the rolling moment caused by the induction, which primarily apply to elliptic wings only. These are wings, the chord of which plotted against the span, gives a half ellipse. With them, and assuming the lift to be proportional to the effective angle of attack, this factor of diminution depends on the aspect ratio only. The same factor can be used approximately for any wings having the same aspect ratio.

The main formulas of the wing theory are:

Induced drag of a wing

$$D_i = \frac{L^2}{k^2\ b^2\ \pi\ V^2\ \frac{\rho}{2}}$$

Mean induced angle of attack

$$\alpha_i = \frac{L}{k^2\ b^2\ \pi\ V^2\ \frac{\rho}{2}}$$

Factor of lift reduction

$$\frac{1}{1 + \frac{2S}{b^2}}$$

Factor of reduction of the rolling moment $\dfrac{1}{1 + \frac{T}{4b^2}}$

Induced yawing moment M_Y due to the rolling moment M_R

$$M_Y = M_R \frac{C_L S}{b^2}$$

where

D_i = the induced drag

α_i = the induced angle of attack

L = the lift

C_L = the lift coefficient $\dfrac{L}{S V^2 \frac{\rho}{2}}$

S = the entire wing area

T = the moment of inertia of the wing area with respect to the axis

b = span

V = velocity of flight

ρ = density of air

k = a factor dependent on the shape of the front view of the wings ($k^2 b^2 \frac{\pi}{4}$ is the area of apparent mass of the front view of the wings).

k = 1 for monoplanes.

(Reference 3.)

4. Propeller Theory.

The assumptions which lead to a practical formula for the efficiency of a propeller, or rather to the upper limit of the efficiency, were first made by Froude. The density of thrust per unit area of the propeller disk is assumed to be constant and the rotation of the slipstream is neglected. The efficiency then has the maximum value compatible with the thrust, the velocity of motion, the diameter of the propeller, and the density of air, and becomes

$$\eta = \frac{2 \times \sqrt{1 + \dfrac{T}{D^2 \frac{\pi}{4} V^2 \frac{\rho}{2}}}}{1 + \sqrt{1 + \dfrac{T}{D^2 \frac{\pi}{4} V^2 \frac{\rho}{2}}}}$$

where

T = thrust

D = diameter

V = velocity

ρ = density

Other information about the properties of propellers is obtained by combining the wing section theory and the slipstream theory of Froude. The blade elements are supposed to act like the wing elements of an ordinary wing, moving along spiral paths. This procedure is rather involved, too, and it seems judicious to simplify it by considering the blades as one unit. The main assumption is that variation of the shape of the slipstream, but not of its velocity v, may be neglected. Then the slipstream velocity, as follows from the consideration of the physical dimension of the quantities determining it, is necessarily a linear function of the velocity of flight and the tip velocity of the blades U. The ratio U/V for zero thrust can be obtained by applying the wing section theory to the blades. It will often be exact enough to consider a mean blade section only, say at 0.7 of the propeller radius, and to find the value of U/V where its lift becomes zero, the air being supposed to be at rest.

The application of the wing section theory in conjunction with the slipstream theory of Froude leads to an approximate formula for the constant differential quotient dv/dU. The choice of $0.7r$ as mean radius of action gives the formula

$$\frac{dv}{dU} = \frac{2.8 \frac{S}{D^2}}{1 + 1.4 \frac{S}{D^2} \left(\frac{U}{V}\right)_0} \qquad \text{(Reference 7)}$$

where $(U/V)_0$ is the value of U/V for zero thrust and S the entire blade area. By means of this formula, and of the relations between the slipstream velocity and the thrust, the thrust can be computed for any value of U/V.

5. Conclusion.

The simplifications of hydrodynamical computations discussed in this paper are of more than practical value for the computation. They are also of great instructive value, as they point out the main causes of the different actions of the air. These are always the same as in rigid mechanics, each force is the reaction to an acceleration of masses. The kinetic energy contained in an air flow, and the momentum giving rise to it are its main characteristics, and play the same part as do the kinetic energy and the momentum of rigid bodies in the mechanics of rigid bodies. These conceptions appeal to the engineer and give rise to creative thoughts. They should therefore be put in the very front in aerodynamical papers intended for education, instead of abstract mathematical conceptions like vortices, which are chiefly of use for special scientific research.

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